Effect of polarized gluon distribution on spin asymmetries for neutral and charged pion production

M. Hirai[1,*] and K. Sudoh[2,†]

[1]*Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization, 1-1, Ooho, Tsukuba, Ibaraki, 305-0801, JAPAN*

[2]*Radiation Laboratory, RIKEN (The Institute of Physical and Chemical Research), Wako, Saitama 351-0198, JAPAN*

(Dated: February 1, 2008)

A longitudinal double spin asymmetry for π^0 production has been measured by the PHENIX collaboration. The asymmetry is sensitive to the polarized gluon distribution and is indicated to be positive by theoretical predictions. We study a correlation between behavior of the asymmetry and polarized gluon distribution in neutral and charged pion production at RHIC.

PACS numbers: 13.85.Ni, 13.88.+e

I. INTRODUCTION

Determination of the polarized parton distribution functions (PDFs) is crucial for understanding the spin structure of the nucleon [1]. As is known well, the proton spin is composed of the spin and orbital angular momentum of quarks and gluons. Several parametrizations of the polarized PDFs have been proposed, and have successfully reproduced experimental data [2, 3, 4, 5, 6]. In particular, the amount of the proton spin carried by quarks is determined well by global analyses with the polarized deep inelastic scattering (DIS) data. The value is about $\Delta\Sigma = 0.1 \sim 0.3$, whereas the prediction from the naive quark model is $\Delta\Sigma = 1$. This surprising result leads to extensive study on the gluon polarization. The current parametrizations suggest a large positive polarization of gluon. However, our knowledge about the polarized gluon distribution $\Delta g(x, Q^2)$ is still poor, since theoretical and experimental uncertainties are rather large. The determination of $\Delta g(x, Q^2)$ gives us a clue to the proton spin puzzle.

The RHIC is the first high energy polarized proton-proton collider to measure $\Delta g(x, Q^2)$ [7]. We can extract information about $\Delta g(x, Q^2)$ through various processes, e.g., prompt photon production, jet production, and heavy flavor production. These processes are quite sensitive to $\Delta g(x, Q^2)$, since gluons in the initial state associate with the cross section in leading order (LO).

Recently, the PHENIX collaboration has reported results for inclusive π^0 production $pp \to \pi^0 X$ [8] which is also likely to be sensitive to $\Delta g(x, Q^2)$. The double spin asymmetry was measured in longitudinally polarized proton-proton collisions at RHIC in the kinematical ranges: center-of-mass (c.m.) energy $\sqrt{s} = 200$ GeV and central rapidity $|\eta| \leq 0.38$. The data imply that the asymmetry might be negative at transverse momentum $p_T = 1 \sim 3$ GeV. The lower bound of the π^0 asymmetry at low p_T has been considered, and a slight negative asymmetry by modifying $\Delta g(x, Q^2)$ has been demonstrated in Ref. [9]. However, there is no theoretical predictions indicating large negative asymmetry.

In this paper, we study the behavior of the π^0 double spin asymmetry correlated with $\Delta g(x, Q^2)$ in Sec II. By using three types of $\Delta g(x, Q^2)$, we suggest that the asymmetry in large p_T region is more sensitive to the functional form of $\Delta g(x, Q^2)$. An impact of the new data on determination of $\Delta g(x)$ is discussed in terms of uncertainty of the asymmetry coming from the polarized PDFs. Furthermore, we discuss a spin asymmetry for charged pion production in Sec. III. An asymmetry taking the difference of cross sections for π^+ and π^- production is proposed, and it is useful to discuss the sign of $\Delta g(x)$ in the whole p_T region. The Summary is given in Sec. IV.

II. SPIN ASYMMETRY FOR NEUTRAL PION PRODUCTION

A. Ambiguity of the polarized cross section

First, we describe the longitudinal double spin asymmetry for π^0 production. It is defined by

$$A_{LL}^{\pi^0} \equiv \frac{[d\sigma_{++} - d\sigma_{+-}]/dp_T}{[d\sigma_{++} + d\sigma_{+-}]/dp_T} = \frac{d\Delta\sigma/dp_T}{d\sigma/dp_T}, \tag{1}$$

where p_T is the transverse momentum of produced pion. $d\sigma_{hh'}$ denotes the spin-dependent cross section with definite helicity h and h' for incident protons.

The cross sections can be separated short distance parts from long distance parts by the QCD factorization theorem. The short distance parts represent interaction amplitudes of hard partons, and are calculable in the framework of perturbative QCD (pQCD). On the other hand, the long distance parts such as PDFs and fragmentation functions should be determined by experimental

*E-mail: mhirai@rarfaxp.riken.jp
†E-mail: sudou@rarfaxp.riken.jp

data. The polarized cross section $\Delta\sigma$ is written by

$$\begin{aligned}\frac{d\Delta\sigma^{pp\to\pi^0 X}}{dp_T} &= \sum_{a,b,c}\int_{\eta^{\min}}^{\eta^{\max}} d\eta \int_{x_a^{\min}}^{1} dx_a \int_{x_b^{\min}}^{1} dx_b \\ &\times\Delta f_a(x_a,Q^2)\Delta f_b(x_b,Q^2) \\ &\times\mathcal{J}\left(\frac{\partial(\hat{t},z)}{\partial(p_T,\eta)}\right)\frac{\Delta\hat{\sigma}^{ab\to cX}(\hat{s},\hat{t})}{d\hat{t}} \\ &\times D_c^{\pi^0}(z,Q^2), \qquad (2)\end{aligned}$$

where $\Delta f_i(x,Q^2)$ is the polarized PDFs, and $D_c^{\pi^0}(z,Q^2)$ is the spin-independent fragmentation function decaying into pion $c\to\pi^0$ with a momentum fraction z. The sum is over the partonic processes $a+b\to c+X$ associated with π^0 production. $\mathcal{J}$ is the Jacobian which transforms kinematical variables from $\hat{t}$ and z into p_T and η of the produced π^0. $\Delta\hat{\sigma}$ describes the polarized cross section of subprocesses. The partonic Mandelstam variables $\hat{s}$ and $\hat{t}$ are defined by $\hat{s}=(p_a+p_b)^2$ and $\hat{t}=(p_a-p_c)^2$ with partonic momentum p_i, respectively. The squared c.m. energy s is related to $\hat{s}$ through $\hat{s}=x_a x_b s$ and set as $\sqrt{s}=200$ GeV. The pseudo-rapidity η is limited as $|\eta|\le 0.38$ in the PHENIX acceptance.

In this analysis, the cross sections and the spin asymmetry are calculated in LO level. Rigorous analysis of $\mathcal{O}(\alpha_s^3)$ next-to-leading order (NLO) calculation has been established in Ref. [10]. We believe that the qualitative behavior of the asymmetry does not change, even if NLO corrections are included in our study. In numerical calculations, we adopt the AAC set [2] as the polarized PDFs and the Kretzer set [11] as the fragmentation functions. We choose the scale $Q^2=p_T^2$.

The partonic subprocesses in LO are composed of $\mathcal{O}(\alpha_s^2)$ $2\to2$ tree-level channels listed as $gg\to q(g)X$, $qg\to q(g)X$, $qq\to qX$, $q\bar{q}\to q(g,q')X$, $qq'\to qX$, and $q\bar{q}'\to qX$ including channels of the permutation $q\leftrightarrow\bar{q}$. Main contribution to the polarized cross section comes from $gg\to q(g)X$ and $qg\to q(g)X$ channels with conventional PDFs and fragmentation functions. The gg contribution dominates in low p_T region and steeply decreases with p_T increases. Then, the qg process becomes dominant in larger p_T region. The crossing point of these contributions however depends on parametrization of the polarized PDFs. In both cases, the spin asymmetry for π^0 production is sensitive to the gluon polarization.

As mentioned above, the partonic cross section $\Delta\hat{\sigma}$ is well-defined in the pQCD framework. Hence, as a cause of inconsistency with the PHENIX data, we consider the ambiguity of long distance parts: fragmentation functions and PDFs.

The fragmentation into π^0 includes all channels $q,\bar{q},g\to\pi^0$. Each component of the fragmentation functions $D_c^{\pi^0}$ can be determined by global analyses with several experiments [11, 12]. The unpolarized cross section measured by the PHENIX [13] are consistent with NLO pQCD calculations within model dependence of $D_c^{\pi^0}$. These precise measurements give strong constraint on the fragmentation functions. Significant modification of them would not be expected. Therefore, the fragmentation functions are not the source of the negative asymmetry even if they have uncertainty to some extent.

In the polarized reaction, kinematical ranges and the fragmentation functions are the same as the unpolarized case except the polarized PDFs. For the polarized quark distributions $\Delta q(x)$ and $\Delta\bar{q}(x)$, the antiquark distributions and their flavor structure are not well known. For π^0 production, subprocesses are (light quark) flavor blind reaction, and the predominant qg process depends on the sum $\Delta q(x)+\Delta\bar{q}(x)$ which is relatively determined well by the polarized DIS data [9], and so ambiguities of the polarized quark distributions can be neglected. In consequence, the undetermined polarized gluon distribution $\Delta g(x)$ remains as the source of the uncertainty of the asymmetry.

B. Correlation between the spin asymmetry and the polarized gluon distribution $\Delta g(x)$

To investigate a role of $\Delta g(x)$ for the behavior of the asymmetry, we prepare three functional forms as shown in Fig 1. Solid curve shows $\Delta g(x)$ by the global analysis with the polarized DIS data [2]. Dashed and dot-dashed curves show two artificial modified $\Delta g(x)$, respectively. The sample-1 distribution has a node. The gluon distribution with a node has been indicated in the paper by Jäger *et al.,* [9]. Our distribution is negative in the small x region, and positive in the large x region. It has opposite signs of $\Delta g(x)$ shown in Fig. 2 of their paper. The sample-2 distribution is small negative in the whole x region. Their distribution is similar to the sample-2 rather than the sample-1. It shows barely positive at small x, while the sample-2 is negative. Since the sample-1 and 2 are within the $\Delta g(x)$ uncertainty by the AAC analysis, these distributions can be adopted as

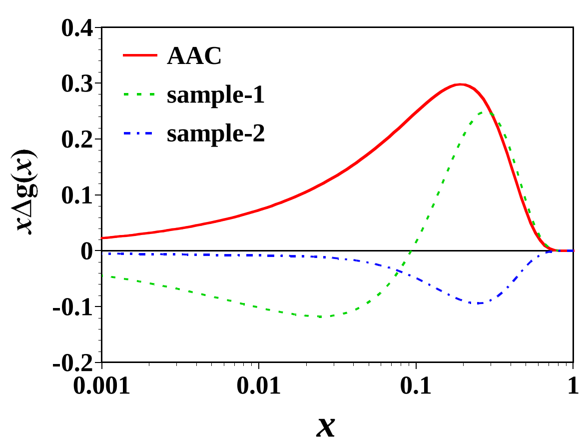

FIG. 1: Polarized gluon distributions $\Delta g(x)$ at $p_T=2.5$ GeV. Solid, dashed, and dot-dashed curves indicate the AAC, sample-1, and 2 distributions, respectively.

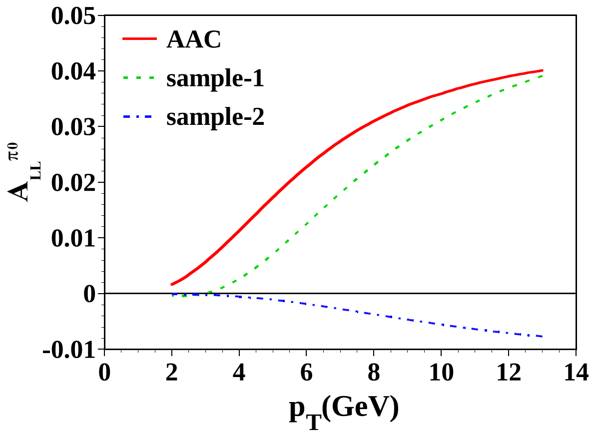

FIG. 2: Spin asymmetries for π^0 production by using three different $\Delta g(x)$ in Fig. 1.

a model of $\Delta g(x)$. These are taken account of the Q^2 dependence by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation with the polarized quark and antiquark distributions.

We discuss the behavior of the spin asymmetry associated with the functional form of $\Delta g(x)$. The obtained asymmetries with these gluon distributions are shown in Fig. 2. We find that the asymmetry for the AAC $\Delta g(x)$ is positive in the whole p_T region. The asymmetries for the sample-1 and 2 become negative at low p_T. In particular, we obtained the negative asymmetry in the whole p_T region by using the sample-2 $\Delta g(x)$. Furthermore, one can see variations of these asymmetries at large p_T.

The asymmetry for the AAC is positive and increases with p_T. The positive polarization for $\Delta g(x)$ generates positive contributions of gg and qg processes which dominate in the π^0 production. In this case, the asymmetry cannot become negative.

The positive $\Delta g(x)$ is suggested by the recent global analyses with the polarized DIS data [2, 3, 4, 5, 6]. Although these analyses obtain good agreement with the experimental data, the $\Delta g(x)$ cannot be determined and it has large uncertainty. Therefore, we cannot rule out the negative polarization for $\Delta g(x)$. There is a possibility of the negative asymmetry with the modified $\Delta g(x)$.

For the sample-1 in Fig. 2, the asymmetry is slight negative at low p_T and changes into positive at $p_T = 3$ GeV. As is mentioned in Ref. [9], the $\Delta g(x)$ with a node has the possibility of making the small negative asymmetry at low p_T. In the region $p_T < 3$ GeV, we find that contributions of gg and qg processes are negative, respectively. To make negative gg contribution would be needed opposite polarizations of $\Delta g(x)$ at x_a and x_b. Computed by using several shapes of $\Delta g(x)$ with a node, the gg contribution is not always negative. The contribution basically depends on the shape of $\Delta g(x)$ even if it has a node.

In the region $p_T > 3$ GeV, the gg contribution changes into positive, and dominates in the region $p_T < 10$ GeV. This is because that the node rapidly shifts toward low-x direction due to Q^2 evolution with p_T. Therefore, the positive polarization for $\Delta g(x)$ at medium x contributes predominantly to the positive asymmetry via the gg process. Furthermore, the asymmetry at large p_T is sensitive to the behavior of $\Delta g(x)$ at medium x.

As another possibility of the negative asymmetry, we choose slight negative polarization for $\Delta g(x)$. In this case, the gg contribution is positive while the qg contribution is negative. The asymmetry is determined by the difference between two contributions. The gg and qg contributions are proportional to $(\Delta g)^2$ and Δg, respectively. The gg contribution is more sensitive to the behavior of $\Delta g(x)$. In particular, the behavior at low x significantly affects on the contribution at low p_T since the value of $x^{\min}$ in Eq. (2) is rather small. In order to make the positive gg contribution smaller, the $\Delta g(x)$ for the sample-2 is taken small polarization at low x as shown in Fig. 1.

In Fig. 2, as far as the sample-2 is concerned, the asymmetry indeed becomes negative in the whole p_T region. In the region $p_T < 3$ GeV, the small negative polarization for $\Delta g(x)$ generates slight positive contribution of the gg process. In this case, the gg contribution is the same order of magnitude as the qg contribution, and almost cancel out the negative contribution. The asymmetry is therefore determined by other processes. The total contribution of the processes except above two processes becomes slight negative. Above the region, the gg contribution rapidly decreases with p_T increases. The qg process becomes dominant contribution, which provides the negative asymmetry [9]. Thus, the negative asymmetry can be obtained in the whole p_T region by using the negative $\Delta g(x)$ which makes the qg contribution larger than the gg contribution.

In the sample-2, we should note that the magnitude of $\Delta g(x)$ at the minimum point cannot be large. This is because that the shape of $\Delta g(x)$ is rapidly varied by the Q^2 evolution, the minimum point of $\Delta g(x)$ shifts toward low-x and the width broadens. At moderate p_T, the gg process is more sensitive to the low-x behavior of the evolved $\Delta g(x)$ than the qg process. If the $\Delta g(x)$ is taken large negative polarization at the minimum point, the magnitude of the gg contribution becomes rapidly large compared with the qg contribution, and then the asymmetry becomes positive at moderate p_T. The small negative $\Delta g(x)$ is therefore required to obtain the negative asymmetry in the whole p_T region.

In above two cases at low p_T, we cannot also obtain negative value exceeded the lower limit -0.1% that is suggested in Ref. [9]. Furthermore, even if the asymmetry is positive, the magnitude is below 1%. As we discussed, the functional form of $\Delta g(x)$ needs some restraints to make the asymmetry negative. It is difficult to obtain sizable value in comparison with the positive case.

At large p_T, the difference of the obtained asymmetries remarkably reflects the medium-x behavior of $\Delta g(x)$. Ex-

perimental data in the region is useful to determine the $\Delta g(x)$. For instance, the asymmetry for the sample-2 becomes rather larger to negative direction. If future precise data indicate the negative asymmetry in the region, the $\Delta g(x)$ requires significant modification of its functional form. It has the potential of the negative gluon contribution to the nucleon spin. In order to understand the behavior of $\Delta g(x)$ in detail, we require experimental data covering a wide p_T region.

C. Uncertainty of the spin asymmetry

Next, we consider the effect of the π^0 data on the $\Delta g(x)$ determination in terms of the uncertainty estimation for the spin asymmetry. The large uncertainty of $\Delta g(x)$ implies the difficulty of extracting the gluon contribution from the polarized DIS data. We are therefore interested in constraint power of the new data on $\Delta g(x)$. If the experimental data are included in a global analysis, the asymmetry uncertainty will be bounded within statistical error range. See, for example, Fig. 2 of Ref. [2]. As far as evaluation of the constraint is concerned, the uncertainty can be compared with statistical errors of the data, although it is rough evaluation.

The asymmetry uncertainty coming from the polarized PDFs is defined by a polarized cross section uncertainty divided by a unpolarized cross section: $\delta A_{LL}^{\pi^0} = \delta\Delta\sigma^{\pi^0}/\sigma^{\pi^0}$. The cross section uncertainty is obtained by taking a root sum square of uncertainties of all subprocesses. These uncertainties are estimated by the Hessian method, and are given by

$$\left[\delta\Delta\sigma_k^{\pi^0}\right]^2 = \Delta\chi^2 \sum_{i,j} \left(\frac{\partial\Delta\sigma_k^{\pi^0}(p_T)}{\partial a_i}\right) H_{ij}^{-1} \left(\frac{\partial\Delta\sigma_k^{\pi^0}(p_T)}{\partial a_j}\right), \tag{3}$$

where k is the index of subprocesses. a_i is a optimized parameter in the polarized PDFs. H_{ij} is the Hessian matrix which has the information of the parameter errors and the correlation between these parameters. The $\Delta\chi^2$ determines a confidence level of the uncertainty, and is estimated so that the level corresponds to the 1σ standard error. We choose the same value as the AAC analysis [2]. Further, the gradient terms for the subprocesses $\partial\Delta\sigma_k^{\pi^0}(p_T)/\partial a_i$ are obtained by

$$\begin{aligned}\frac{d\Delta\sigma_k^{\pi^0}}{dp_T} &= \sum_{a,b,c}\int_{\eta^{\min}}^{\eta^{\max}} d\eta \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 dx_b \\ &\times \left[\frac{\partial\Delta f_a(x_a)}{\partial a_i}\Delta f_b(x_b) + \Delta f_a(x_a)\frac{\partial\Delta f_b(x_b)}{\partial a_i}\right] \\ &\times \mathcal{J}\left(\frac{\partial(\hat{t},z)}{\partial(p_T,\eta)}\right)\frac{\Delta\hat{\sigma}^{ab\to cX}(\hat{s},\hat{t})}{d\hat{t}} D_c^{\pi^0}(z), \end{aligned} \tag{4}$$

The gradient terms for the polarized PDF $\partial\Delta f_a(x_a)/\partial a_i$ are analytically obtained at initial scale Q_0^2, and are numerically evolved to arbitrary scale Q^2 by the DGLAP equation.

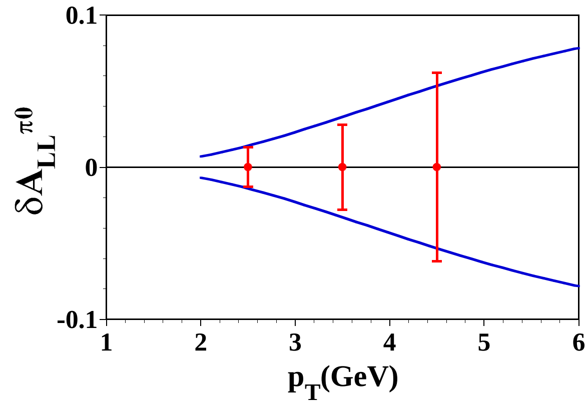

FIG. 3: Comparison of the asymmetry uncertainty $\delta A_{LL}^{\pi^0}$ with the statistical errors for $\sqrt{s} = 200$ GeV.

In Fig. 3, the asymmetry uncertainty is compared to the statistical errors of the experimental data by the PHENIX [8]. In this comparison, we exclude the data at $p_T = 1.5$ GeV. This is because that the data might have contribution from soft physics, and it might not be explained as physics of a hard process. We have no idea whether such data can be included in the global analysis. From this figure, we find that the uncertainty almost corresponds to the experimental errors, and is mainly composed of the uncertainty of $\Delta g(x)$. This fact indicates that the present π^0 data have the same constraint on $\Delta g(x)$ as the polarized DIS data. At this stage, one cannot expect to reduce the $\Delta g(x)$ uncertainty even if these data are included into the global analysis. However, the asymmetry uncertainty is very sensitive to the $\Delta g(x)$ uncertainty. The π^0 production has the potential to become a good probe for $\Delta g(x)$ by future precise data.

It should be noted that symmetric uncertainty is shown in order to compare with the statistical errors in Fig. 3. The lower bound is however incorrect because a lower limit of the asymmetry is not taken into account. As mentioned in previous subsection, the asymmetry cannot exceed -0.1 % at low p_T where the gg process dominates. Although asymmetric uncertainty should be estimated, such uncertainty cannot be obtained by the Hessian method. We therefore need further investigation of the lower bound for the asymmetry uncertainty.

III. SPIN ASYMMETRY FOR CHARGED PION PRODUCTION

We discuss the spin asymmetry for charged pion production, π^+ and π^-. Unpolarized and polarized cross sections can be similarly calculated by using the fragmentation functions decaying into charged pion $D^{\pi^\pm}$ in Eq. (2). We show asymmetries with the AAC $\Delta g(x)$ and sample-2 $\Delta g(x)$ in Fig. 4. In the asymmetries for the AAC $\Delta g(x)$, one can see differences among them in large p_T region where the qg process is dominant. The

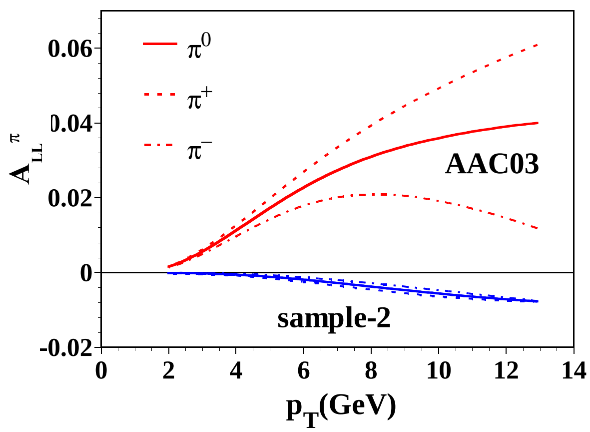

FIG. 4: Asymmetries for neutral and charged pion productions with the AAC and sample-2 $\Delta g(x)$ sets.

polarized cross sections of qg process for π^+ and π^- production are written by

$$\Delta\sigma_{qg}^{\pi^\pm} = \Delta g \otimes \left(\sum_i \Delta f_i \otimes D_i^{\pi^\pm}\right) \otimes \Delta\hat{\sigma}^{qg\to qg} + \Delta g \otimes \left(\sum_i \Delta f_i\right) \otimes D_g^{\pi^\pm} \otimes \Delta\hat{\sigma}^{qg\to qg} \ . \quad (5)$$

where the symbol $\otimes$ denotes convolution integral in Eq. (2). i indicates the quark flavor, and is taken as $i = u, d, s, \bar{u}, \bar{d}$, and $\bar{s}$. Actual calculation includes permutated terms of x_a and x_b. There are following relations among the fragmentation functions for charged pion:

$$D_u^{\pi^+} > D_u^{\pi^-}, \ D_d^{\pi^+} < D_d^{\pi^-}, \quad D_q^{\pi^+} = D_{\bar{q}}^{\pi^-}, \ D_g^{\pi^+} = D_g^{\pi^-}, \quad (6)$$

and the fragmentation functions for neutral pion are defined by

$$D_i^{\pi^0} = (D_i^{\pi^+} + D_i^{\pi^-})/2 \ . \quad (7)$$

For π^+ production, the contribution associated with Δu distribution is enhanced by the fragmentation function $D_u^{\pi^+}$. Increasing asymmetry for π^+ production is caused by positive contribution from Δu distribution, whereas decreasing asymmetry for π^- production comes from negative Δd distribution.

On the other hand, the asymmetries for the sample-2 $\Delta g(x)$ are almost the same. The differences among them depends on the magnitude of $\Delta g(x)$, since the asymmetry is proportional to $\Delta g(x)$ as written in Eq. (5). If the absolute value of $\Delta g(x)$ is small, there are not significant differences among the asymmetries for π^0, π^+, and π^- productions.

In order to determine $\Delta g(x)$ with its sign by using charged pion production, let us propose an interesting observable which is defined by

$$A_{LL}^{\pi^+-\pi^-} = \frac{\Delta\sigma^{\pi^+-\pi^-}}{\sigma^{\pi^+-\pi^-}} \equiv \frac{\Delta\sigma^{\pi^+} - \Delta\sigma^{\pi^-}}{\sigma^{\pi^+} - \sigma^{\pi^-}} \ . \quad (8)$$

The behavior of the asymmetry is sensitive to the sign of $\Delta g(x)$ because the contribution of the gg processes are eliminated and one of the qg process becomes dominant in the whole p_T region. The polarized cross section for $gg \to gg$ process is given by

$$\Delta\sigma_{gg}^{\pi^\pm} = \Delta g \otimes \Delta g \otimes D_g^{\pi^\pm} \otimes \Delta\hat{\sigma}^{gg\to gg}. \quad (9)$$

This contribution is cancelled out due to $D_g^{\pi^+} = D_g^{\pi^-}$. For the same reason, $gg \to q\bar{q}$ process does not also contribute by summing fragmentation functions for flavors: $\sum_i D_i^{\pi^+} = \sum_i D_i^{\pi^-}$. As the similar case, the contributions of $q\bar{q} \to gg$, $q\bar{q} \to q'\bar{q}'$ processes are also vanished. The unpolarized cross section can be similarly calculated with unpolarized PDF's and partonic cross sections.

The asymmetry can be obtained by the difference of qg process. The second term of Eq. (5) is cancelled out for the same reason of $gg \to gg$ process. And then, the asymmetry is consequently given by

$$A_{LL}^{\pi^+-\pi^-} \simeq \frac{\Delta g \otimes (\Delta u_v - \Delta d_v) \otimes (D_1^\pi - D_2^\pi) \otimes \Delta\hat{\sigma}^{qg\to qg}}{g \otimes (u_v - d_v) \otimes (D_1^\pi - D_2^\pi) \otimes \hat{\sigma}^{qg\to qg}} \ , \quad (10)$$

where $\Delta f_v (= \Delta f - \Delta\bar{f})$ is a polarized valence quark distribution. The following relations among the fragmentation functions are assumed by the isospin symmetry,

$$\begin{cases} D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} = D_{\bar{u}}^{\pi^-} = D_d^{\pi^-} \equiv D_1^\pi \\ D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} = D_d^{\pi^+} \equiv D_2^\pi \end{cases} \quad (11)$$

This relation is used in parametrization of the fragmentation functions [11].

In the asymmetry in Eq. (10), ambiguity of fragmentation function $D_g^{\pi^\pm}$ is removed by the cancellation of the convolution part. Another ambiguity from the fragmentation functions can be also cancelled between numerator and denominator. In addition, $\Delta u_v - \Delta d_v$ is determined well, since its first moment is constrained by neutron and hyperon beta decay constants [2, 3, 4, 5]. Of course, unpolarized PDF's are precisely determined in comparison with the polarized PDF's. This asymmetry can be defined by well known distributions without Δg; therefore, we can effectively extract information about $\Delta g(x)$ including its sign.

Figure. 5 shows the asymmetry defined by Eq. (8). Solid and Dotted curves are asymmetries with AAC $\Delta g(x)$ and $-\Delta g(x)$. We find large asymmetries in both cases. In particular, the asymmetry with $-\Delta g(x)$ is negative and the absolute value is large in comparison with single pion production. Since the asymmetry is dominated by qg process in the whole p_T region, the difference of the sign of $\Delta g(x)$ is markedly reflected in the asymmetry.

We mention the contribution of qq process to the asymmetry. In the region $8 < p_T < 13$ GeV, the qq contribution accounts for 10-15% of the polarized part $(\Delta\sigma^{\pi^+} - \Delta\sigma^{\pi^-})$, and 27-56% of the unpolarized part

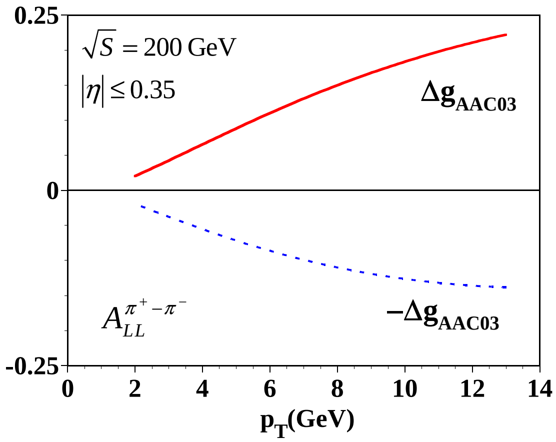

FIG. 5: Asymmetries for the difference of charged pion production with $\Delta g(x)$ and $-\Delta g(x)$.

$(\sigma^{\pi^+} - \sigma^{\pi^-})$ of the asymmetry in Eq. (8). These contributions are not negligible. In particular, effect of the qq contribution in the polarized part appears as the difference between the absolute values of asymmetries. Contributions of all sub-processes are taken into account, however the $q\bar{q}^{(\prime)} \to q\bar{q}^{(\prime)}$ and $qq' \to qq'$ processes are negligible. The difference is due to the positive contribution of qq process. The asymmetry with $-\Delta g(x)$ is therefore suppressed.

Next, we evaluate the experimental sensitivity of this spin asymmetry. We compare the statistical error of the asymmetry $\delta A_{LL}^{\pi^+-\pi^-}$ with one of π^0 production $\delta A_{LL}^{\pi^0}$. An expected statistical error $\delta A_{LL}^{\pi^+-\pi^-}$ is given by

$$\delta A_{LL}^{\pi^+-\pi^-} = \frac{1}{P^2}\frac{1}{\sqrt{N^{\pi^+}}}\frac{\sqrt{1+\alpha}}{1-\alpha}, \tag{12}$$

where P is the beam polarization. α is the ratio of the number of event for π^- and π^+: $\alpha = N^{\pi^-}/N^{\pi^+}$. $N^{\pi^\pm}$ are obtained by the integrated luminosity $\mathcal{L}$ and the unpolarized total cross section $\sigma^{\pi^\pm}$: $N^{\pi^\pm} = \mathcal{L}\sigma^{\pi^\pm}$. The ratio of these statistical errors can be obtained by

$$R_{sta} \equiv \frac{\delta A_{LL}^{\pi^+-\pi^-}}{\delta A_{LL}^{\pi^0}} = \frac{1}{\sqrt{2}}\frac{1+\alpha}{1-\alpha}\ . \tag{13}$$

The parameter α has energy dependence, and decreases with p_T increases.

Table. I represents the value of these parameters. $R_{asym} \equiv A_{LL}^{\pi^+-\pi^-}/A_{LL}^{\pi^0}$ is the ratio of asymmetries. The $\pi^+ - \pi^-$ asymmetry is about 5 times larger than the π^0 asymmetry. In the region $p_T < 11$ GeV, the statistical error becomes larger than the rate of the asymmetry. The constraint power of experimental data are weaker than that of the π^0 asymmetry below the region. However, the value of R_{asym} becomes larger than R_{sta} above the region. The asymmetry would have the same impact on $\Delta g(x)$ as the π^0 production. Although more luminosity is needed in comparison with π^0 production, it is useful to determine effectively the behavior of $\Delta g(x)$ with the sign.

TABLE I: The value of parameters α, R_{sta}, and R_{asym}.

p_T (GeV)	9	10	11	12	13
α	0.82	0.80	0.78	0.76	0.74
R_{sta}	7.2	6.4	5.7	5.2	4.7
R_{asym}	5.0	5.1	5.3	5.5	5.6

IV. SUMMARY

In summary, we have investigated the correlation between the behavior of the spin asymmetry for pion production and the functional form of $\Delta g(x)$. The experimental data by the PHENIX indicates the negative asymmetry at low p_T, and motivate us to modify the functional form of $\Delta g(x)$ drastically. In order to obtain negative asymmetry, the functional form of $\Delta g(x)$ requires some restraints. By modifying $\Delta g(x)$, the slight negative asymmetry can be obtained at low p_T. Moreover, we have indicated the existence of the negative polarization of $\Delta g(x)$ which keeps the asymmetry to be negative in the whole p_T regions. The large negative asymmetry is inconsistent with the theoretical predictions by using $\Delta g(x)$ from polarized DIS data. However, experimental uncertainties are large at present. It is premature to conclude that the pQCD framework is not applicable to π^0 production in polarized pp collisions.

Uncertainty of the π^0 asymmetry coming from the polarized PDF's with DIS data is correspond to the current statistical errors by the PHENIX. These data have the same constraint power on $\Delta g(x)$ as present DIS data. The future measurements will provide useful information for clarifying the gluon spin content.

Furthermore, we have proposed the spin asymmetry defined by the difference of cross sections for π^+ and π^- production. We have discussed an impact of the asymmetry on determination of $\Delta g(x)$. In the asymmetry $A_{LL}^{\pi^+-\pi^-}$, the gg processes are cancelled out, and qg process becomes dominant. Ambiguity of the fragmentation functions can be reduced. The behavior of the asymmetry is sensitive to the sign of $\Delta g(x)$. One can obtain new probe for $\Delta g(x)$ in pion production at RHIC.

[1] For a review see: B. Lampe and E. Reya, Phys. Rept. **332**, 1 (2000).

[2] Asymmetry Analysis Collaboration, M. Hirai, S. Kumano, and N. Saito, Phys. Rev. D **69**, 054021 (2004).

[3] J. Blümlein and H. Böttcher, Nucl. Phys. B **636**, 225 (2002).
[4] E. Leader, A. V. Sidorov, and D. B. Stamenov, Eur. Phys. J. C **23**, 479 (2002).
[5] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D **63**, 094005 (2001).
[6] D. de Florian and R. Sassot, Phys. Rev. D **62**, 094025 (2000).
[7] For example, G. Bunce, N. Saito, J. Soffer, and W. Vogelsang, Annu. Rev. Nucl. Part. Sci. **50**, 525 (2000).
[8] PHENIX Collaboration, S. S. Adler *et al.*, hep-ex/0404027, to be published in Phys. Rev. Lett.
[9] B. Jäger, M. Stratmann, S. Kretzer, and W. Vogelsang, Phys. Rev. Lett. **92**, 121803 (2004).
[10] B. Jäger, A. Schäfer, M. Stratmann, and W. Vogelsang, Phys. Rev. D **67**, 054005 (2003); D. de Florian, Phys. Rev. D **67**, 054004 (2003).
[11] S. Kretzer, Phys. Rev. D **62**, 054001 (2000), and references therein.
[12] B. A. Kniehl, G. Kramer, and B. Pötter, Nucl. Phys. B **582**, 514 (2000).
[13] PHENIX Collaboration, S. S. Adler *et al.*, Phys. Rev. Lett. **91**, 241803 (2003).